This exam consists of 5 numbered pages.

ERASMUS UNIVERSITY ROTTERDAM
Erasmus School of Economics
International Bachelor Economics & Business Economics, Bachelor 1

Exam Mathematics 1 (FEB11003X-10)

Friday 29 October 2010, 9:30 – 12:30 hrs

Instructions ESE

- You are not allowed to use a calculator.
- You are not allowed to use a programmable calculator.
- You are not allowed to use notes (except a cheat sheet, see below).
- You are not allowed to use books.
- You are not allowed to use a dictionary.
- You are allowed to take the examination papers with you (but you have to turn in the separate answer sheet).

Additional information

You are allowed to use a cheat sheet, for which the following rules apply:

- Two-sided A4
- Your name and student number in the upper right corner

All material that does not satisfy these rules will be taken away from you and may be considered a fraud attempt.

Use the separate answer sheet to indicate your answers.

The exam consists of 19 problems grouped in four parts, each with different types of problems. For the basic problems, multiple choice problems and calculation problems you will score 3, 4 and 5 points, respectively, per correct answer and no points for incorrect answers. For the open problems (8 points per problem) your score will depend on the answer and the calculation. The exam grade is the result from the formula (10 + number of scored points)/10. So 63 scored points result in grade 7.3, since (10+63)/10= 7.3.

Good luck!

Part I: Basic problems

3 points per problem

Problem 1

Suppose $f(x)$ is an even function (i.e., a function of which the graph is symmetric about the y-axis). Now consider the function $f(x+d)$, where d is some positive constant. Is it possible that this function is also even? Explain your answer.

Problem 2

Suppose that the function f with domain D has an inverse function f^{-1}. Does this imply that f is either strictly increasing or strictly decreasing on D? Explain your answer.

Problem 3

Let f be a function that is differentiable on its domain. Give an example (i.e., you have to specify a specific function) that shows that a stationary point of f is not necessarily an extreme point.

Problem 4

We know that a continuous function with a domain that is a closed and bounded interval always has a minimum and a maximum. Show that a function that is not continuous does not necessarily have a minimum and a maximum on a closed and bounded interval.

Part II: Multiple choice problems

4 points per problem

Problem 5

Solve $\frac{1}{9}3^{2x^2+4x} = \left(3^x\right)^{2x}$ for x. Which of the following answers is correct?

A. $x = 0$ or $x = 2$ B. $x = \frac{1}{2}$ C. $x = 2$ D. $x = 0$ or $x = \frac{1}{2}$

Problem 6

The derivative of the function $f(x) = (3x+1)\left(\frac{1}{x^2} + \frac{1}{x}\right)$ is

A $-\frac{4x+2}{x^3}$

B. $-3\left(\frac{2}{x^3} + \frac{1}{x^2}\right)$

C. $\frac{3}{x^2} + \frac{3}{x}$

D. $-\frac{2}{x^3} - \frac{7}{x^2} - \frac{3}{x}$

Problem 7

Calculate the derivative of $f(x) = (\ln x + x)^2$. The answer is:

A. $2\left(\frac{\ln x}{x} + x\right)$

B. $2\left(\frac{1}{x} + x\right)$

C. $2(\ln x + x)\left(\frac{1}{x} + 1\right)$

D. $2\left(\frac{1}{x} + 1\right)$

Problem 8

Let $f(x) = \frac{2x}{x^2 + 2}$. The interval on which this function is increasing is

A. $[0, \sqrt{2}]$

B. $(-\infty, \sqrt{2}]$

C. $[-\sqrt{2}, \sqrt{2}]$

D. $[\sqrt{2}, \infty)$

Problem 9

Let $f(x,y) = \sqrt{\ln(x^2) - \ln(y^2)}$, where $x > 0$ and $y > 0$. Calculate the partial elasticity of *f* with repect to *y*. Which of the following answers is correct?

A. $\frac{1}{\ln(x^2) - \ln(y^2)}$

B. $-\frac{2}{\ln(x^2) - \ln(y^2)}$

C. $\frac{1}{\ln x - \ln y}$

D. $-\frac{1}{2\ln x - 2\ln y}$

Problem 10

Consider the function $F(x,y,z,) = \left(\sqrt{x} + \frac{2x}{\sqrt{y}} + 7\frac{y}{\sqrt{x}}\right)^4$. The degree of homogeneity is

A. $\frac{1}{2}$

B. 1

C. 2

D. 4

Part III: Calculation problems

5 points per problem

Problem 11

Determine the remainder of the polynomial division $\frac{x^3 - 12x^2 - 42}{x - 3}$.

Problem 12

Consider the function $y = 3\ln(x-3) + 7$ with domain [4,∞). Determine the inverse and its domain. If the inverse does not exist, report 'no inverse'.

Problem 13

Consider the equation $5xy + 3y^2 - x^2 y = 7$. Use implicit differentiation to compute the value of y' in the point $(x, y) = (1,1)$.

Problem 14

Consider a demand function *D*(*p*) that gives demand as function of price *p*. Suppose that when *p*=100, demand is equal to 350 and the price elasticity of demand is equal to -12. What will (approximately) be the level of demand when price *p*=101?

Problem 15

Find the maximum and minimum value of the function $f(x) = x^2 + \frac{16}{x}$ on the interval [1, 3].

Problem 16

Let $f(x, y) = 2xy + y^x + \ln(3x^2 + 1)$. Find all first- and second-order partial derivatives.

Part IV: Open problems

8 points per problem

Problem 17 (2+2+2+2=8 points)

Consider the function $f(x) = x^5 - 5x^4 + 5x^3 + 3$.

a) Determine the first derivative and the stationary point(s) of f.

b) Classify the stationary point(s) using the first derivative (local minimum, local maximum or not an extreme point).

c) Determine the second derivative f'' and all value(s) of x for which $f''(x) = 0$.

d) Determine on which interval(s) the function f is concave. (It may be useful to know that $\sqrt{2} \approx 1.4$, $\sqrt{3} \approx 1.7$, $\sqrt{5} \approx 2.2$ and $\sqrt{7} \approx 2.6$)

Problem 18 (2+2+1+3=8 points)

Consider the function $f(x, y) = x^2y^2 - 4y^2 - 9x^2 + 49$.

a) Calculate all first order partial derivatives.

b) Calculate all second order partial derivatives.

c) Find all stationary points of f.

d) Calculate the value of f in every stationary point and use the second-derivative test to classify them (local minimum/maximum, saddle point or inconclusive).

Problem 19 (2+3+3=8 points)

We want to find the maximum and minimum values of $f(x, y) = 5x^2y$ subject to $g(x, y) = x^2 + y^2 = 9$. Note that the minimum and maximum values exist, because f is a continuous function and the constraint curve is closed and bounded (it is a circle).

a) Give the Lagrangian function and the first order conditions of the Lagrange multiplier method for this particular problem.

b) Find the point(s) (x,y) and the corresponding value(s) of λ that satisfy the first order conditions of the Lagrange multiplier method.

c) What are the maximum and minimum values and at which point(s) are they attained?