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ABC-formula:

$ax^2+bx+c=0$

$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Simplify:

$x^a x^b = x^{a+b}$

$(x^a)^b = x^{ab}$

$x^a y^a = (xy)^a$

$\sqrt{x} = x^{1/2}$

$\sqrt{xy} = \sqrt{x} * \sqrt{y}$

$x = \log_a y \to a^x = y$

$\log_e x = \ln x$

$x = \ln y \to e^x = y$

$a^x = b \to x = \frac{\ln b}{\ln a}$

$\ln ab = \ln a + \ln b$

$\ln \frac{a}{b} = \ln a - \ln b$

$\ln a^b = b \ln a$

Derivatives:

$f(x)$	$f'(x)$	$f''(x)$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	$\frac{-1}{4x\sqrt{x}}$
e^x	e^x	e^x
e^{ax}	ae^{ax}	a^2e^{ax}
$\ln x$	$\frac{1}{x}$	$\frac{-1}{x^2}$
$\log_a x$	$\frac{1}{\ln a}*\frac{1}{x}$	$\frac{1}{x^2 \ln a}$
a^x	$a^x \ln a$	$a^x(\ln a)^2$

Sum/Difference:

$f(x)\pm g(x) \to f'(x)\pm g'(x)$

Productrule:

$f(x)g(x) \to f'(x)g(x)+f(x)g'(x)$

Quotient:

$f(x)/g(x) \to \frac{f'(x)g(x)-f(x)g'(x)}{g(x)^2}$

Chain rule:

$f(g(x)) \to f'(g(x))*g'(x)$

exp & log functions:

$e^{\ln x} = x$

$\ln e^x = x$

$e^0 = 1$

$1 < e^x < \infty \to for: x > 0$

$0 < e^x < 1 \to for: x > 0$

$\ln 1 = 0$

$\ln e = 1$

$0 < \ln x < \infty$ for: $x > 1$

$-\infty < \ln x < 0$ for $0 < x < 1$

$\ln x$ doesn't exist for $x < 0$

Shifting functions:

Original: $f(x)$

$f(x+d)$ shifts d to the left

$f(x-d)$ shifts d to the right

$f(x)+a$ shifts a up

$f(x)-a$ shifts a down

$c*f(x)$ if c>0, stretched vertically with intensity c

$c*f(x)$ if c<0, above and mirrored around x-axis

$f(-x)$ mirrored around y-axis

$\ln x$ exists for x > 0

$\sqrt{x}$ exists for x> 0

Inverse:

If $y = f(x)$, retrieve x so you get $x = f(y)$

Domain inverse = range $f(x)$

Range inverse = domain $f(x)$

Domain: all values possible for x

Range: all possible values of f(x)

Elasticity:

Perceptual change of the function value if a variable changes with 1 percent.

$EL_x f(x) = \frac{x}{f(x)} * f'(x)$

$EL_y f(y) = \frac{y}{f(y)} * f'(y)$

Limits:

$\lim_{x\to c} \frac{f(x)}{g(x)}$

If limit goes to real number: simplify and plug it in.

L'Hôpital's rule conditions:

-f and g are differentiable

-inserting c would give 0 / $\pm\infty$

- $\lim_{x\to c} \frac{f'(x)}{g'(x)}$ is defined

Then you can use L'Hôpital:

$\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$

Continuity:

$f(x)$ is continuous if:

$\lim_{x\to a} f(x) = f(a)$

Conditions for continuity:

$f(x) = \frac{a}{b} \to b \neq 0$

$f(x) = \sqrt{a} \to a \geq 0$

$f(x) = \ln b \to b > 0$

Rising, falling, convex, concave

Rising: $f'(x) \geq 0$

Falling: $f'(x) \leq 0$

Convex: $f''(x) \geq 0$

Concave: $f''(x) \leq 0$

Partial derivative:

$f'x(x,y)$ derivative wrt x

$f'y(x,y)$ derivative wrt y

$f'xx(x,y)$ twice wrt x

$f'yy(x,y)$ twice wrt y

$f'xy(x,y)$ first x, then y

$f'xy(x,y) \equiv f'yx(x,y)$

Max, min & stationary:

1 variable:

Stationary when $f'(x) = 0$

max: $f''(x) < 0$

min: $f''(x) > 0$

Multiple variables:

Stationary: $f'_x = 0$ **&** $f'_y = 0$

Strictly local extreme:

$f''_{xx} f''_{yy} - f''^2_{xy} > 0$

max: $f''_{xx} < 0$

min: $f''_{xx} > 0$

Saddle point when:

$f''_{xx} f''_{yy} - f''^2_{xy} < 0$

Inflection points:

Where the function changes, convex or concave:

$f''(x) = 0$

Homogenities:

$f(x,y,z)$ has a degree of homogeneity k if function $f(tx,ty,tz)$ can be rewritten as

$t^k * f(x,y,z)$ Multiply every variable with t, and get it out.

Lagrange:

Maximize $f(x,y) = cx + dy$ subject to the restriction $ax + by = m$

Lagrange function is:

$L = cx + dy - \lambda(ax + by - m)$

Calculate L'_x and L'_y

$L'_\lambda = ax + by - m$

To calculate x and y:

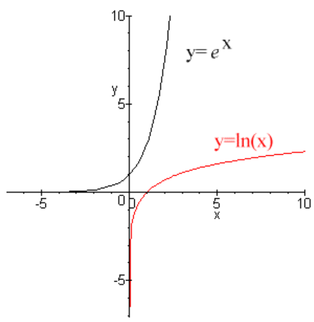
$L'_y = L'_x$

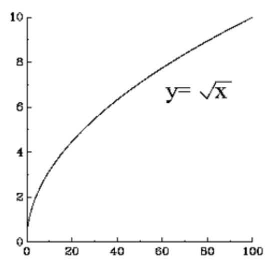
Take out common factors.

When you have y= $f(x)$, calculate x & y with restriction

To calculate λ, fill in values for x or y into L'_x or L'_y.

In economics, λ is the shadowprice, and shows the change of f when value of the restriction changes with 1 unit.





Implicit differentiation:

Calculate derivatives while treating the other variable as a function of x, so for a derivative make it y'.

$f(x,y): xy^2 = 3$

$x[y(x)]^2 = 3$

Using product and chain rule:

$f'_x = y^2 + 2xy \cdot y'$

Asymptotes:

we have a *vertical* asymptote for the values of x when the denominator is equal to zero:

$\frac{(x-c)(x+d)}{(x-a)(x+b)}$

vertical asymptotes then are: $x = a$ and $x = -b$

horizontal asymptotes:

Degree of numerator = x

Degree of denominator = z

If z > x, hor. asymptote: $y = 0$

If x > z, hor. asymp doesn't exist

If x = z, hor. asymptote is:

$\frac{\text{numerator's leading coefficient}}{\text{denominator's leading coefficient}}$

Oblique asymptote:

$y(x) = \frac{x^2+3x-4}{x-2}$

Perform polynomial division and disregard the remainder, result gives the oblique asymptote: $x+5$