Mathematics 1 FEB11003X
Model exam

This model exam consists of problems that have been part of actual exams in the past. It has the same structure as the exam that you will take. There are four parts, each with different types of problems. A maximum of 90 points can be scored. The grade for this written exam is the result from the formula (10 + number of scored points)/10. All problems should (and can) be made without the use of an electronic calculator.

Part I Basic Problems

3 points per problem

Problem 1

Explain whether or not it is necessary that a function $f(x)$ is defined at x=a when we want to compute $\lim_{x \to a} f(x)$.

Problem 2

Describe in words the meaning of the elasticity of a function $f(x)$ with respect to x.

Problem 3

Consider the function $f(x,y) = (x+y)^2$. What can you say about the sign (≤ 0 or ≥ 0) of f''_{11}, f''_{22} and $f''_{11} \cdot f''_{22} - (f''_{12})^2$? Is this function concave, convex, or neither?

Problem 4

Consider a function $f(x,y)$ where both x and y are non-negative. Explain whether or not such a function has always a maximum and a minimum on its domain.

Part II Multiple Choice Problems

4 points per problem

Problem 5

Solve $2^{3t}4^t = 16$ for t. Which of the following answers is correct?

A. $\frac{1}{2}$ B. $\frac{4}{5}$ C. 1 D. $\frac{2}{3}$

Problem 6

Given a function $f(x)$ and its inverse function $g(x)$. Which of the following statements is correct?

A. $f(g(x)) = 1$ B. $g(f(x)) = 1$ C. $f(g(x)) = x$ D. $f(g(x)) = \frac{1}{x}$

Problem 7

Find the elasticity $El_x y$ of $y = x^3 e^{2x+1}$. Which of the following answers is correct?

A. $3+2x$ B. $3+x$ C. $\frac{3}{x}+2$ D. $2+3x$

Problem 8

Consider the function $f(x,y) = x^3 - x^2 - 3x + 2xy^2 - y^4$.

The number of stationary points is:

A. 0 B. 1 C. 2 D. 3

Problem 9

$f(x,y) = \sqrt{\ln x^2 + \ln y}$

Calculate the partial elasticity of f with respect to x and the partial elasticity of f with respect to y. Divide the first by the second.

Which of the following answers is correct?

A. $\frac{2y}{x}$ B. 2 C. $\frac{1}{x}$ D. $\frac{y}{x^2}$

Problem 10

Consider the function $f(x,y) = x^3 - 2xy + y^3$.

Which classification is true for the point $(0,0)$:

A. local minimum B. local maximum C. saddle point D. miscellaneous (i.e. the 2nd order test is not conclusive)

Part III Calculation Problems

5 points per problem

Fractions, logarithms and powers of e do not have to be (numerically) calculated.

Problem 11

Compute the value of x for which $\frac{7}{x+7} - \frac{5}{x^2-49} = \frac{6}{x-7}$.

Problem 12

Compute $\lim_{x\to 3} \frac{x^2+x-12}{x-3}$.

Problem 13

Compute $g''(1)$ if $g(t) = e^{2t^2+\ln t}$ (the second derivative of g in $t = 1$).

Problem 14

Find the maximum and minimum of the function $f(x) = \frac{x^2+1}{x}$ on the interval $[\frac{1}{2}, 2]$.

Problem 15

Consider the function $f(x, y, z) = xy + yz + xz$, with $x = r + s, y = r - s$, and $z = rs$. Calculate the first order derivative of f with respect to r as well as the first order derivative of f with respect to s at the points $r = 1$ and $s = 1$.

Problem 16

Consider the function $f(x, y) = 2x^2 + 3y^2$ subject to $y - x = 5$.

Find the stationary points (there could also be none or one) of the Lagrangian function $\mathcal{L}(x, y)$ and the corresponding values of λ.

Part IV Open Problems

8 points per problem

Fractions, logarithms and powers of *e* do not have to be (numerically) calculated.

Problem 17

Calculate all stationary points for the function $f(x) = x^5 - 5x^4 + 5x^3 + 3$. Determine for each stationary point whether it is a local maximum, a local minimum or an inflection point.

Problem 18

Given the function $f(x, y) = x^2y^2 - 4y^2 - 9x^2 + 36$.

(a) Calculate all first order partial derivatives.

(b) Calculate all second order partial derivatives.

(c) Find all stationary points of f.

(d) Calculate the value of f in every stationary point and use the second-derivative test to classify them (local minimum, local maximum, saddle point, miscellaneous (i.e. the 2nd order test is not conclusive)).

Problem 19

Find all points (there may be only one or none) that correspond to a local maximum of the function $f(x, y) = \frac{1}{3}x^3 + 2y^2$ subject to the constraint $y + x = 3$.