This exam consists of 5 numbered pages.

ERASMUS UNIVERSITY ROTTERDAM

Faculty of Economics
International Bachelor Economics & Business Economics, Bachelor 1

Exam Mathematics 1 (FEB11003X-07)

Wednesday, October 24, 2007, 9:30 – 12:30 hrs

This is a closed book exam. Electronic calculators are not allowed (and not needed).

You are allowed, however, to use a cheat sheet, for which the following rules apply:

- Two-sided A4
- Your name and student number in the right upper corner
- Handwritten in your own handwriting
- No photo copies

All material that does not satisfy these rules will be taken away from you and may be considered a fraud attempt.

Use the separate answer sheet to indicate your answers. The exam consists of four parts, each with a different type of problem. For the basic problems, multiple choice problems and calculation problems you will score 3, 4 and 5 points, respectively, per correct answer and no points for incorrect answers. For the open problems (8 points per problem) your score will depend on the answer and the calculation. The exam grade is the result from the formula (10 + number of scored points)/10. So 63 scored points results in (10+63)/10= 7.3.

Good Luck!

Part I: Basic Problems

3 points per problem

Problem 1

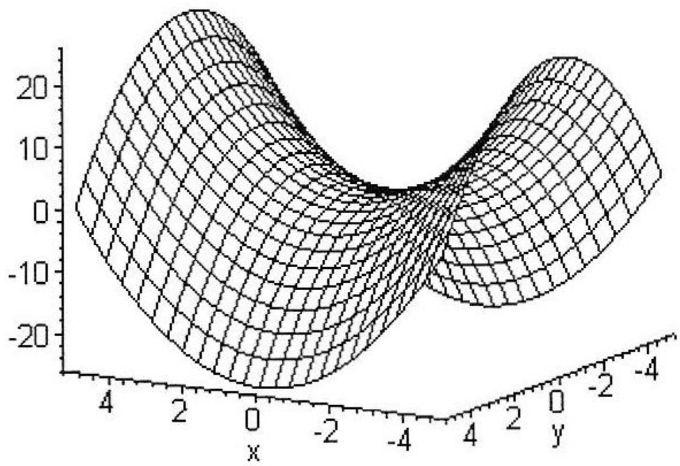
Describe the difference between the concepts derivative and elasticity in words.

Problem 2

Let $y = f(x)$ be a function. In the context of differentiation, mention two situations in which it is useful to consider $\ln y$ instead of y.

Problem 3

Consider the graph of a function with variables x and y in the following figure. Is this function convex, concave or neither of the two?



Problem 4

Let (x_0, y_0) be a stationary point of the function $f(x, y)$ on a convex domain D. Then (x_0, y_0) is a minimum point if for all (x, y) it holds that $f''_{11}(x, y) > 0$, $f''_{22}(x, y) > 0$ and $f''_{11}(x, y)f''_{22}(x, y) - (f''_{12}(x, y))^2 > 0$. Explain why the condition $f''_{22}(x, y) > 0$ does not have to be mentioned explicitly.

Part II: Multiple Choice Problems

4 points per problem

Problem 5

Consider the graph of the function $y = f(x)$. If we replace the graph of $y = f(x)$ by the graph of $y = f(x - c)$ with $c > 0$, then the graph of $f(x)$ shifts

A. c units to the left B. c units to the right C. c units upwards
D. c units downwards

Problem 6

Let $f(x, y) = \sqrt{\ln(x^2) - \ln(y^2)}$. Calculate the partial elasticity of f w.r.t. x, calculate the partial elasticity of f w.r.t. y, and add them up. Which of the following is the answer?

A. -1 B. $\frac{1}{\ln x - \ln y}$ C. $\frac{2}{\ln x - \ln y}$ D. 0

Problem 7

Let $f(x, y) = x^3 + 2xy + \frac{1}{4}y^4 - \ln y$. Find the values of the following derivatives in the point $(1, 1)$ and add them up: f'_x, f'_y, f''_{xx}, f''_{xy}, f''_{yx}, and f''_{yy}. Which of the following answers is correct?

A. 18 B. 19 C. 21 D. 22

Problem 8

Determine the first derivative of y (y') in the point $(3, 2)$ of $2x^2 + xy - y^2 = -4$ by implicit differentiation. Which of the following answers is correct?

A. 2 B. $\frac{10}{3}$ C. $\frac{17}{4}$ D. 14

Problem 9

Let $f(K, L) = K^2 + L^2 - 2KL$ with $K = \ln(t + 1)$ and $L = e^{3t}$. Calculate the derivative of $f(K, L)$ w.r.t. t for $t = 0$. Which of the following answers is correct?

A. 0 B. 2 C. 4 D. 6

Problem 10

Solve $e^{1-t^2} = 4$. Which of the following answers is correct?

A. $t = \pm\sqrt{1 + \ln 4}$ B. $t = \pm\sqrt{1 - \ln 4}$ C. $t = 1 \pm \sqrt{\ln 4}$
D. $t = -1 \pm \sqrt{\ln 4}$

Part III: Calculation Problems

5 points per problem

Problem 11

Compute the value(s) of x that satisfy the following equation: $5^{x^2+28x-63} = 125^{9x-7}$.

Problem 12

Solve for x: $\frac{7}{x+7} - \frac{5}{x^2-49} = \frac{6}{x-7}$.

Problem 13

Find the maximum and minimum point of the function $f(t) = \frac{27}{t} + 3t + 7$ on the interval $[1, 27]$.

Problem 14

Find the inverse and its domain of the function $y = 3 - 4e^{x+2}$.

Problem 15

Compute the degree of homogeneity of the function $F(x, y, z) = \left(\sqrt{x} + \frac{2x}{\sqrt{y}} + 2\frac{z}{\sqrt{y}}\right)^4$.

Problem 16

Consider the function $F(x, y) = xy$ with $x = e^t$ and $y = \ln t$. Compute the derivative of $F(x, y)$ w.r.t. t.

Part IV: Open problems

8 points per problem

Problem 17 $(2+2+2+2$ points)

Consider the function $f(x) = x^6 - 10x^4$.

a. Calculate the first derivative and the stationary point(s).

b. Classify the stationary point(s) using the first derivative (local minimum, local maximum or not an extreme point).

c. Calculate the second derivative $f''(x)$ and the point(s) for which $f''(x) = 0$.

d. Classify the point(s) from part c using the second derivative (inflection point or not an inflection point).

Problem 18 $(2+2+2+2$ points)

Given the function $f(x,y) = x^3 - 3xy + y^3$.

a. Calculate the first order partial derivatives of f.

b. Calculate the second order partial derivatives of f.

c. Calculate the stationary points of f.

d. Calculate the function value of f for every stationary point and use the second order test to classify them (local minimum, local maximum, saddle point), if possible.

Problem 19 $(2+2+2+2$ points)

Assume we want to maximize or minimize the function $f(x,y) = 3x + y$ subject to the constraint $g(x,y) = x^3y = 81$.

a. Give the Lagrangian function.

b. Give the first order conditions of the Lagrange multiplier rule.

c. Calculate the point(s) that satisfy the first order conditions.

d. What is the economic interpretation of the value of λ?